

International Journal of Modern Physics A
 © World Scientific Publishing Company

CONSTRAINTS ON PARTON DISTRIBUTIONS AND THE STRONG COUPLING FROM LHC JET DATA

JUAN ROJO

*Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road,
 University of Oxford, OX1 3NP Oxford, United Kingdom
 Juan.Rojo@physics.ox.ac.uk*

Received Day Month Year

Revised Day Month Year

Jet production at hadron colliders provides powerful constraints on the parton distribution functions (PDFs) of the proton, in particular on the gluon PDF. Jet production can also be used to extract the QCD coupling $\alpha_s(Q)$ and to test its running with the momentum transfer up to the TeV region. In this review, I summarize the information on PDFs and the strong coupling that has been provided by Run I LHC jet data. First of all, I discuss why jet production is directly sensitive to the gluon and quark PDFs at large- x , and then review the state-of-the-art perturbative calculations for jet production at hadron colliders and the corresponding fast calculations required for PDF fitting. Then I present the results of various recent studies on the impact on PDFs, in particular the gluon, that have been performed using as input jet measurements from ATLAS and CMS. I also review the available determinations of the strong coupling constant based on ATLAS and CMS jet data, with emphasis on the fact that LHC jet data provides, for the first time, a direct test of the $\alpha_s(Q)$ running at the TeV scale. I conclude with a brief outlook on possible future developments.

Keywords: Jet physics; QCD; Parton Distributions; Strong Coupling.

PACS numbers: 13.87.-a, 12.38.Bx, 12.38.Qk

1. Introduction

The backbone of global fits of parton distribution functions (PDFs) is provided by deep-inelastic scattering (DIS) measurements from fixed-target experiments and from the HERA collider. While DIS data provides stringent constraints on the quark PDFs, it is only indirectly sensitive to the gluon PDF through the scaling violations encoded in the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations.^{1–3} Thanks to the available lever arm, at not too small values of Bjorken- x these scaling violations and the precise inclusive HERA data^{4,5} allow a reasonably accurate determination of the gluon PDF. However, at medium and large values of x , the gluon is virtually unconstrained from DIS data, and thus affected by large uncertainties.

For this reason, global PDF fits require complementary measurements that are directly sensitive to the gluon PDF in the medium and large- x region. When the

first precision measurements of jet production at Run I at the Tevatron^{6–8} were released, it became apparent that it should be possible to access the large- x gluon using differential distributions in jet production. In addition, it was recognized that a proper estimate of the PDF uncertainties was essential for any search for New Physics at large transverse energies (E_T) involving jets in the final state.^{9,10} With this motivation of constraining the poorly known large- x gluon, the latest inclusive jet measurements from the Tevatron Run II^{11,12} are included in most global PDF fits.^{13–15}

While for quite some time Tevatron jet measurements were the only ones available, since a few years a plethora of jet production data at the Large Hadron Collider (LHC) is allowing us to test our understanding of perturbative Quantum Chromodynamics (QCD) to an unprecedented level of precision, and to provide unique constraints on the gluon and quark PDFs at large- x .^a Jet measurements from ATLAS and CMS are available from the 2010, 2011 and 2012 data taking periods of Run I, either in a final or in a preliminary format, and several of these datasets have already been used in PDF studies. These measurements cover a wide kinematic range in jet transverse momentum, dijet invariant mass, and rapidity, and provide precious information on the large- x PDFs.

In addition, the LHC jet data have made possible a number of determinations of the strong coupling constant α_s , and allowed for the first time to test its running at the TeV scale. The running of α_s is dictated by the renormalization group (RG) evolution equations, and could be modified as compared to the Standard Model (SM) predictions in the presence of New Physics beyond the SM, for example for new colored sectors. While available determinations are typically limited by the scale uncertainties from the next-to-leading order (NLO) calculations, recent progress on the full next-to-next-to-leading order (NNLO) corrections suggest that such theory errors will be substantially reduced in the near future.

In this review, I review the constraints that jet measurements from the LHC have provided on the proton PDFs, specially on the gluon, and on the strong coupling constant. The structure of this review is the following. I begin in Sect. 2 with a discussion of the PDF sensitivity of jet production. Then in Sect. 3 I review the theoretical calculations and tools that allow us to extract information on PDFs and α_s from the collider jet data. I continue in Sect. 4 presenting available studies that quantify the impact of LHC jet data on the PDFs, and in Sect. 5 I summarize available determinations of the strong coupling from LHC jet production. I conclude in Sect. 6 and discuss the outlook for future measurements and studies based on the LHC jet data.

Let us recall also that jet production is not the only final state through which the gluon PDF and α_s can be probed at the LHC. Related complementary processes include isolated photon production,¹⁷ top quark pair production^{18,19} and high- p_T Z

^aA detailed review of the constraints on PDFs obtained at Run I has been presented in the PDF4LHC report¹⁶

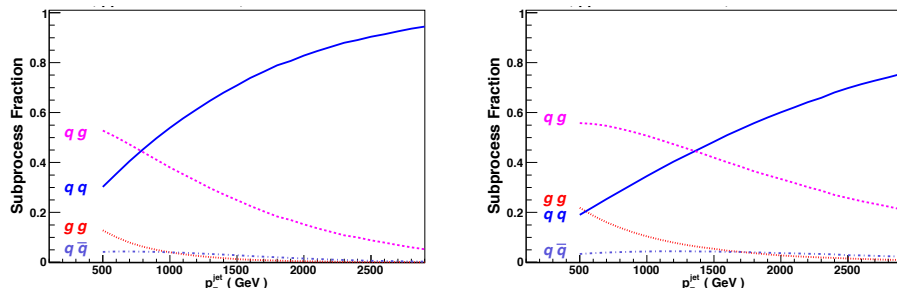


Fig. 1. Subprocess decomposition of inclusive jet production at the LHC 8 TeV (left plot) and 14 TeV (right plot), computed with ALPGEN at LO using MSTW08 as input PDFs, as a function of the p_T of the leading jet.

boson production.²⁰ In particular, top quark pair production benefits from reduced theory uncertainties from the recent NNLO calculation,²¹ though it cannot compete with jet production in terms of kinematical reach for the PDFs. For the small- x gluon, it has been recently showed how important information can be obtained from the LHCb charm production data.^{22,23}

2. PDF sensitivity of jet production in hadronic collisions

To quantify the PDF coverage of LHC jet data, it is useful to review the kinematics of jet production at hadron colliders. For simplicity, one can work in the Born approximation, where the leading processes are of the form

$$\text{parton}_i(p_1) + \text{parton}_j(p_2) \rightarrow \text{parton}_k(p_3) + \text{parton}_l(p_4),$$

with i, j, k, l flavor indices to denote either quarks, antiquarks or gluons. In Fig. 1 I show the subprocess decomposition of inclusive jet production at the LHC 8 TeV (left plot) and 14 TeV (right plot), computed with ALPGEN²⁴ at leading order using MSTW08¹⁴ as input PDFs.

From Fig. 1 we see that at 8 TeV qg scattering is the main production mechanism for $p_T^{\text{jet}} \leq 800$ GeV, then qq scattering becomes more important due to the steeper fall-off of the gluon PDF at large- x . At the LHC 14 TeV, gluon-initiated contributions dominate up to $p_T^{\text{jet}} \sim 1.5$ TeV. Therefore, it is clear that there is a wide range in jet p_T for which gluon-initiated contributions dominate the cross-section, and thus measurements in this range allow to pin down the gluon PDF. At the highest values of p_T , jet production instead probes the large- x quarks. Note also that the contributions initiated by antiquarks are much smaller since their PDFs are strongly suppressed as compared to those of quarks and gluons at large- x .

Denoting by y_3 and y_4 the rapidities of the two outgoing partons in the laboratory reference frame, and with p_T being their (back-to-back) transverse momentum, one finds that dijet production probes the PDFs at the following values of their mo-

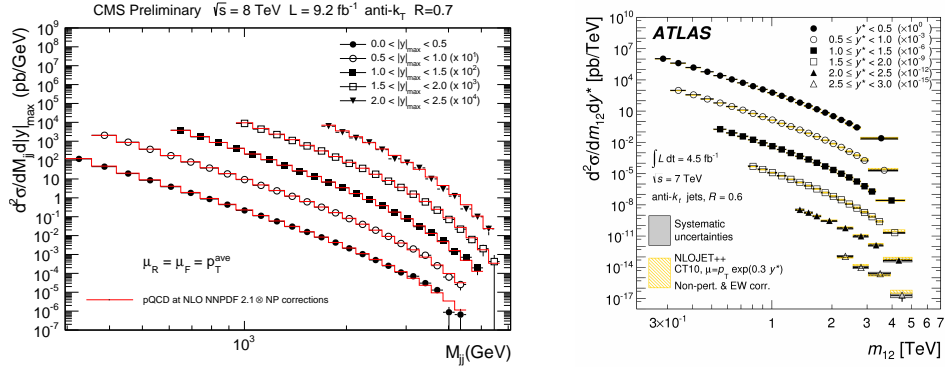
4 *Juan Rojo*

Fig. 2. Left plot: preliminary CMS data²⁵ for dijet production at 8 TeV. The measured cross-sections are compared to NLO QCD theory, using NNPDF2.1 as input PDFs. Right plot: results for the ATLAS 2011 dijet measurement,²⁶ now using CT10 as input PDF. In both cases the theoretical calculations include non-perturbative and electroweak corrections.

momentum fractions:

$$x_1 = \frac{p_T}{\sqrt{s}} (e^{y_3} + e^{y_4}), \quad x_2 = \frac{p_T}{\sqrt{s}} (e^{-y_3} + e^{-y_4}),$$

with \sqrt{s} being the center-of-mass energy of the hadronic collision. Note that in inclusive jet production instead the underlying values of x_1 and x_2 are not uniquely defined in terms of the measurement final state kinematics, usually taken to be p_T^{jet} and y_{jet} . Dijet production is typically measured as a function of the dijet invariant mass m_{34} , which in the Born kinematics is given by

$$m_{34} = 2 p_T \cosh\left(\frac{y_3 - y_4}{2}\right) \equiv 2 p_T \cosh y^*,$$

which is manifestly invariant under longitudinal boosts, and where we have defined the rapidity difference of the two outgoing partons as $y^* \equiv (y_3 - y_4)/2$. Dijet production measurements are typically binned in m_{34} and in y^* , and an upper limit on the maximum rapidity of the individual jets $|y_{3,4}| \leq y_{\text{max}}$ is imposed. It is thus possible to derive the range of Bjorken- x probed in dijet production, in terms of the measured final state kinematics, which is given by

$$x_{\min} = \frac{m_{34}}{\sqrt{s}} e^{-y_{\text{max}} + y^*} \leq x \leq 1.$$

Therefore in dijet production we can have access to PDFs with Bjorken- x from x_{\min} to one. Similar expressions can be derived for inclusive jet production measurements.

As illustrative examples of the kinematical coverage of LHC jet data, in Fig. 2 (left plot) I show the preliminary CMS data²⁵ on dijet production at 8 TeV, where the results are compared to NLO QCD theory using NNPDF2.1²⁷ as input PDFs. Note that the reach in the dijet invariant mass m_{34} is almost 6 TeV. Then in Fig. 3 I show the value of the x_{\min} probed in the PDFs for dijet production at 7 TeV, using the kinematics of the ATLAS dijet measurement from the 2011 dataset.²⁶

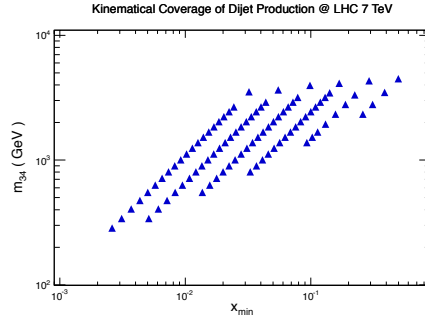


Fig. 3. Minimum value of Bjorken- x and the scale m_{34} probed in the PDFs for dijet production at the LHC 7 TeV, using the kinematics of the ATLAS 2011 dijet measurement.²⁶

The corresponding cross-section measurements are shown in the right plot of Fig. 2. As can be seen, in this particular case the dijet data probes the PDFs in the range $x \gtrsim 2 \cdot 10^{-3}$ and for momentum transfers in the range of $2 \cdot 10^2 \lesssim Q \lesssim 5 \cdot 10^4$ GeV. In addition, the higher the invariant mass of the dijet system m_{34} , the larger the value of Bjorken- x that will be probed.

While Fig. 3 determines the region of Bjorken- x that is kinematically accessible in jet production measurements, it does not provide information on which part of this region dominates the production cross-section, or in other words, the region of Bjorken- x for which the PDF sensitivity of the jet data is maximized. To determine this important information, it is possible to compute the correlation coefficients between the PDFs and the experimental data. As explained in Ref.,²⁸ in a Monte Carlo PDF set one can compute the correlation between the parton distributions, for different values of x and Q^2 , and the jet production cross-sections, for different bins of jet transverse momentum and rapidity.

Using NNPDF2.1 NLO, this exercise was carried out in the CMS analysis of Ref.,²⁹ which studies the constraints on PDFs and on α_s of their 7 TeV inclusive jet data. The results can be found in Fig. 4, which shows the correlation coefficient between PDFs (in this case the gluon and the up quark) for all the p_T bins in the central rapidity region, $|y| \leq 0.5$, as a function of Bjorken- x and the momentum transfer Q . A value of this coefficient close to one (minus one) indicates that, for this specific data bin, the cross-section is strongly (anti-)correlated with the corresponding PDFs in the given range of x . In particular, from Fig. 4 one can see that LHC inclusive jet data has a strong correlation with the gluon for $x \geq 0.1$, with a likewise strong anti-correlation for $x \sim 10^{-2}$. This correlation is weaker for the up quark, except for large values of x , that is, $x \gtrsim 0.4 - 0.5$, for which the qq scattering channel begins to dominate over qg scattering, see Fig. 1.

3. Theory calculations and tools for fitting jet data

The NLO cross-sections for jet production at hadron colliders have been known for a long time.^{30,31} They have been implemented in various computer programs,

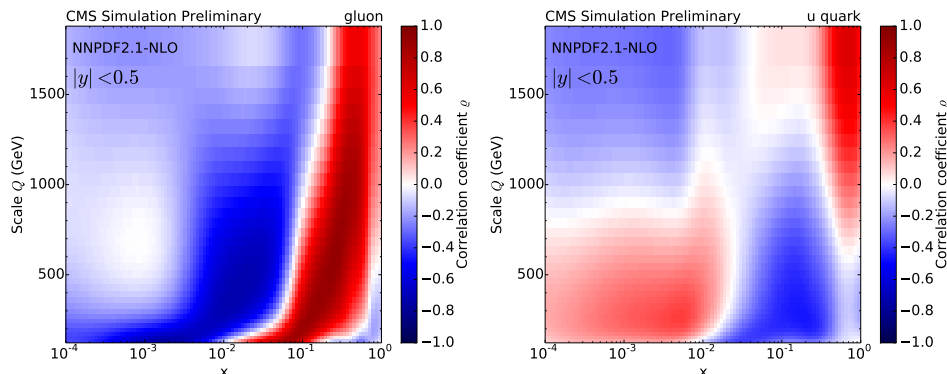


Fig. 4. Correlation coefficients between the gluon PDF (left plot) and the up quark PDF (right plot) for different values of (x, Q^2) for the kinematics of the CMS 7 TeV inclusive jet cross-section. The computation of these coefficients, performed with NNPDF2.1 NLO, has been performed in all the jet p_T bins of the central rapidity region, $|y| \leq 0.5$. Results taken from Ref.,²⁹ additional figures are available from <https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSMP12028>.

such as NLOJET++.³² Computing differential distributions for jet observables with these codes is however very CPU-time intensive, and thus not suitable for the aims of PDF determinations, where the iterative fitting procedure requires recomputing the same observables a large number of times. With this motivation, different fast interfaces to NLO jet calculations have been developed. The basic idea of these interfaces is to precompute the partonic cross-section in a grid in (x_1, x_2, Q^2) , so that it is possible to perform an a posteriori convolution with the PDFs to yield the hadronic cross-section much more efficiently. In particular, the NLOJET++ calculations have been interfaced to both APPLGRID³³ and FASTNLO,³⁴ making possible to include collider jet data into PDF fits without any K-factor approximation. More recently, the AMCFast interface³⁵ to MADGRAPH5_AMC@NLO³⁶ has been released. AMCFast also allows to include NLO jet calculations into PDF fits, with the additional possibility of accounting also for the matching of the fixed-order calculation to parton showers.

Going beyond NLO, thanks to recent breakthroughs in our ability to perform computations at NNLO in QCD for processes that include colored particles both in the initial and final state,³⁷ the full NNLO cross-section for inclusive jet and dijet production in the gg channel has recently become available.^{38,39} This result is an important milestone towards the full NNLO calculation. The ongoing work in the remaining partonic channels suggests that the full NNLO cross-section for hadronic jet production will be available in the near future. These results are of paramount importance to extend the physics potential of the interpretation of LHC jet measurements, often limited by the theoretical systematics from the NLO scale variations.

While the full NNLO calculation of jet production becomes available, it is possi-

ble to include jet data in NNLO fits by using approximate NNLO calculations based on threshold resummation, such as those derived in Refs.^{40–42}. The most recent of these calculations,⁴⁰ among other improvements, includes the full dependence on the jet radius R in the resummed result. However, before being able to use these approximate NNLO calculations in a PDF fit, it is crucially important to determine the range of validity of the threshold approximation. This can be done by comparing the exact gg NNLO calculation with the approximate NNLO in the same partonic channel, using identical cuts and binning as those of the corresponding Tevatron and LHC jet measurements.

This idea has been exploited in Ref.⁴³ which provides the complete list of approximate NNLO K-factors, and their region of validity, for all published Tevatron and LHC inclusive jet production measurements. This is all the information that is needed in order to include jet data into a NNLO fit. In particular, Ref.⁴³ finds that while the threshold approximation is good at high p_T and central rapidities, it is much poorer at low p_T and forward rapidities. Therefore, it is possible to include most of the available LHC jet measurements in a global NNLO PDF fit provided that one restricts the fitted data to the central and high- p_T regions. This is the strategy that has been used in the NNPDF3.0 fits.⁴⁴

Finally, one should mention that at the high transverse momenta and invariant masses that the LHC is and will be covering, electroweak corrections to jet production are non negligible. For example, the calculation of purely weak radiative corrections for dijet production at hadron colliders⁴⁵ finds corrections that can be as large as 10% at the highest possible jet transverse momenta. While accounting for these effects is probably not essential for PDF fits to Run I jet data, given the experimental uncertainties in the high p_T region, their inclusion will become mandatory with the Run-II data, which will explore deep into the TeV region.

4. PDF constraints from LHC jet data

Using the theoretical calculations and the fast interfaces that I have discussed in the previous section, it becomes possible to include LHC jet data into PDF fits and quantify the constraints that it provides. First of all, I present the PDF studies that have been performed by the ATLAS and CMS collaborations. From the ATLAS side, the available measurements relevant for PDF studies are the inclusive jet and dijet cross sections from the 2010 dataset,⁴⁶ the ratio of 2.76 TeV to 7 TeV inclusive jet cross-sections⁴⁷ and the dijet cross-sections from the 2011 dataset.²⁶

The first ATLAS QCD analysis of jet data was performed using their measurement ratio of jet cross-sections between two different center-of-mass energies. The motivation for such a ratio measurement is that due to the cancellation of several theoretical (in particular scale variations) and experimental (jet energy scale) uncertainties, there is a complementary PDF sensitivity as compared to the absolute cross-section measurements.⁴⁸ In Ref.⁴⁷ this PDF analysis of the ratio of 2.76 TeV over 7 TeV inclusive jet data was performed to demonstrate the sensitivity to the

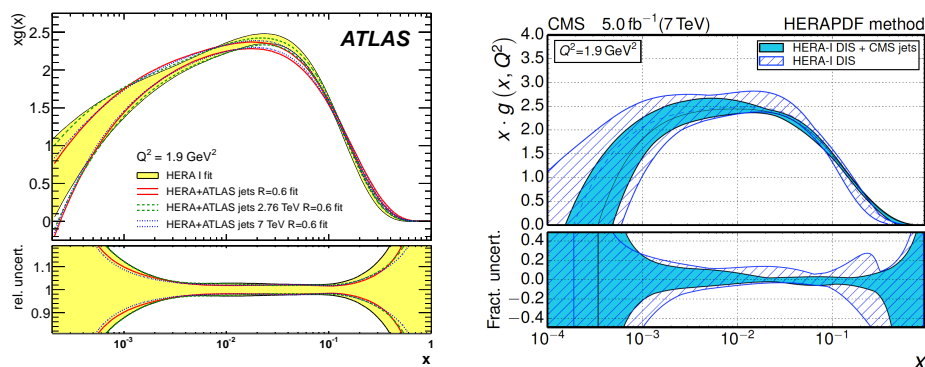


Fig. 5. Representative results of PDF analyses using jet data obtained by the ATLAS and CMS collaborations. Left plot: impact on the gluon PDF of the ATLAS data on the 2.76/7 TeV inclusive jet cross-section ratio.⁴⁷ Right plot: constraints on the gluon from the CMS inclusive jet 2011 data.²⁹ In both cases the results of the PDF fit are compared to a HERA-only baseline fit.

large- x gluon PDF, by comparing to a baseline fit based on HERA data only. The results of this analysis, based on the HERAFITTER framework, are shown in Fig. 5. The reduction on the gluon PDF large- x uncertainties can be appreciated.

More stringent constraints can be provided by the ATLAS jet measurements from the 2011 run. The inclusive jet cross-section from the 2011 run has been presented in Ref.⁴⁹, where it is compared to several modern PDF sets. The ATLAS dijet analysis from the 2011 data²⁶ contains a quantitative χ^2 comparison for the predictions of different PDF sets, showing a clear discrimination power. In addition, the ATLAS measurement of three-jet production cross-sections⁵⁰ from the 2011 data can also be potentially useful for PDF studies. These measurements are particularly interesting for PDF fits since for the first time the full correlation matrix between inclusive jet, dijet and three-jet data is publicly available, allowing to include in a PDF fit the complete ATLAS jet production dataset from 2011.

CMS has also studied the PDF sensitivity of their jet production measurements. The quantification of the PDF constraints from the inclusive jet cross sections measured in the 2011 dataset⁵¹ has been studied by CMS in Ref.²⁹ As expected, a substantial reduction on the medium and large- x gluon PDF uncertainties as compared to a baseline HERA-only fit is found, as illustrated in Fig. 5. This CMS analysis also emphasizes the crucial role of a careful estimation of systematic uncertainties and their correlation to improve the PDF sensitivity of jet measurements. In this CMS study the traditional analysis based on HERAFITTER is complemented with a Monte Carlo analysis with data-based regularization. Ongoing measurements of inclusive jets⁵² and dijets²⁵ at 8 TeV will further extend the constraining power of the CMS measurements.

While not directly usable in PDF fits, the CMS measurement of the ratio of jet cross-sections at different values of the jet radius, $R = 0.5$ and $R = 0.7$, see Ref.,⁵³

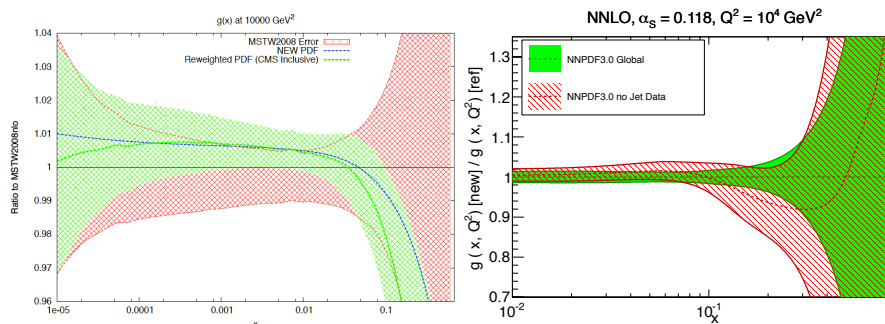


Fig. 6. Left plot: the impact of the CMS inclusive jet data from the 2011 dataset on the MSTW08 gluon.⁵⁶ Right plot: the improvement in the large- x gluon PDF uncertainties thanks to the inclusion of Tevatron and LHC jet data in the NNP3.0 NNLO fit.⁴⁴

provides useful information on the theory systematics that affect these measurements specially at moderate p_T .^{54,55} In this respect, note that ATLAS jet measurements are provided for two different jet radii, $R = 0.4$ and $R = 0.6$, see for example Ref.,⁴⁶ allowing similar studies as those of the CMS paper, and in particular providing an important cross-check that the impact of jet data on PDFs does not depend strongly on the specific value of R .

In addition to these studies performed by the LHC collaborations, most global PDF fitting groups have explored the impact of the LHC jet data, and some already include them in their most recent releases. The MSTW collaboration has presented a detailed analysis of the impact of LHC inclusive jet and dijet data into their PDFs,⁵⁶ restricted to NLO theory. As can be seen in Fig. 6, a reduction of the PDF uncertainties at large- x from the CMS inclusive jet data is reported, with the new gluon tending to be softer than the baseline gluon in MSTW08. Ref.⁵⁶ also finds difficulties in achieving a good χ^2 for the dijet data, and improving the situation might require the full NNLO calculation. See⁵⁷ for updated studies in the context of the MMHT14 global analysis.

The LHC jet data from ATLAS and CMS is also part of the recent NNP3.0 fit, and is included both in the NLO and NNLO fits, in the latter case using the approximate NNLO calculations as discussed in Ref.⁴³ In Fig. 6 a variant of the NNP3.0 fit without jet data is compared to the global fit baseline. The results that NNP3.0 finds are consistent with those reported by MSTW, in particular for substantial error reduction that LHC data provides. The CT collaboration has also compared their predictions to LHC jet data in Ref.¹⁵, measurements that will be included in the future CT14 release. Studies of the impact of hadron collider jet data in the ABM framework have been reported in Ref.⁵⁸

To conclude this section, it should be clear from the above discussion that PDF fits with LHC jet data have been so far restricted to inclusive jet measurements, excluding dijet data. One important practical reason for this is that typically the correlation between the two datasets is unknown, and thus dijets are not included

to avoid double counting.^b However, the main reason is that dijet calculations are affected by larger theoretical uncertainties than inclusive jets, as reflected by the fact that the wide choice of scales allowed by dijet kinematics leads to quite different cross-sections.⁵⁶ In this respect, the completion of the NNLO dijet calculation should make possible to include these data in PDF fits without further problems.

5. Determinations of $\alpha_s(M_Z)$ from LHC jets

Now I turn to review the status of the determinations of the strong coupling constant from LHC data. The motivation of such determinations is three-fold. First of all, the uncertainty on the strong coupling constant $\alpha_s(M_Z)$ is a substantial contribution to the total theoretical uncertainty in important LHC processes, such as Higgs production in gluon fusion.^{28,59,60} Second, accurate $\alpha_s(M_Z)$ measurements allow to test the possible unification of the strong, weak and electromagnetic coupling constants at very high scales.⁶¹

Finally, the energy reach available at the LHC makes possible for the first time to perform direct measurements of $\alpha_s(Q)$ in the TeV scale, and thus provide model-independent constraints on Beyond the Standard Model scenarios which are characterized by a different running of the coupling as compared to QCD above a certain mass scale, see for example Refs.^{62–64} Indeed, various studies based on LHC data provide determinations both of the value of the strong coupling evolved down to the Z boson mass, $\alpha_s(M_Z)$, using the RG equations, as well as direct extractions of $\alpha_s(Q)$ for $Q \gg M_Z$ that allow to test its running with the scale Q and to compare with the extrapolations of the SM predictions.

Within the framework of global PDF analyses, jet production measurements provide a crucial handle, allowing a reliable extraction of $\alpha_s(M_Z)$. In particular, jet measurements from the Tevatron in global fits play an instrumental role in stabilizing the gluon, and in turn the closely related value of $\alpha_s(M_Z)$,^c as discussed for instance in Refs.^{60,65,66} In particular, the effect of the LHC jet data in the best-fit $\alpha_s(M_Z)$ in the MSTW framework has also been studied.⁵⁶ In global PDF analyses that include jet data, the value of $\alpha_s(M_Z)$ that is extracted is typically consistent with the PDG average, which in its most updated version⁶⁷ reads

$$\alpha_{\text{PDG}}(M_Z) = 0.1185 \pm 0.0006.$$

On the other hand, extractions of $\alpha_s(M_Z)$ from PDF fits without jet data⁶⁸ tend to be systematically lower.

Concerning direct determinations of α_s from jet measurements at the LHC, a number of analyses have been presented, mostly by the ATLAS and CMS Collaborations themselves. Currently all these extractions of the strong coupling are

^bAs mentioned above, this will be no longer the case with the upcoming ATLAS 2011 jet measurements.

^cThis is so because from deep-inelastic scattering data only it is difficult to separate the effects of a change on the gluon PDF from those of a variation of α_s .

restricted to NLO accuracy, and therefore the scale uncertainties from perturbative higher orders are the main limiting factor of the resulting accuracy. It is clear that, once the full NNLO results are available, future reanalysis of these determinations of α_s will substantially reduce the associated theory uncertainties. In the meantime, using the approximate NNLO calculations following the recipe of Ref.⁴³ might provide a way forward to reduce these dominant theoretical uncertainties.

The CMS collaboration presented its first extraction of $\alpha_s(M_Z)$ from the measurement of the ratio of three-jets over two-jet cross-sections⁴⁷ based on the 7 TeV 2011 data. This ratio is directly proportional to $\alpha_s(Q)$, where Q is defined as the average transverse momentum of the two leading jets, that is,

$$Q = \langle p_{T,1,2} \rangle \equiv \frac{p_{T1} + p_{T2}}{2}.$$

One important advantage of this ratio is the partial cancellation of experimental and theoretical systematic uncertainties common to the three-jet and the two-jet cross-sections. The result of this analysis is

$$\alpha_s(M_Z) = 0.1148 \pm 0.0014 (\text{exp}) \pm 0.0018 (\text{PDF}) \pm 0.0050 (\text{th}),$$

using the NNPDF2.1 as input PDF set.^d The precision is thus limited by the QCD scale uncertainties from the NLO calculation. In addition to the extraction of $\alpha_s(M_Z)$, separate determinations of α_s in bins of $\langle p_{T,1,2} \rangle$ are also provided, including the first direct determination of the strong coupling constant in the ~ 1 TeV range. These extractions of $\alpha_s(Q)$ in the TeV scale provide model-independent constraints on new physics, which predict a different running with the scale as compared to the SM. For instance, Ref.⁶² uses this CMS measurement of $R_{3/2}$ to provide model-independent constraints on new sectors of colored matter.

As mentioned in the previous section, in Refs.²⁹ the CMS collaboration studied the impact on the PDFs of their 2011 7 TeV inclusive jet production data.⁵¹ In the same analysis, CMS also extracted the strong coupling, and their best-fit result turns out to be

$$\alpha_s(M_Z) = 0.1185 \pm 0.0019 (\text{exp}) \pm 0.0028 (\text{PDF}) \pm 0.0004 (\text{NP})^{+0.0053}_{-0.0024} (\text{scale}),$$

where again the precision of the determination of the coupling constant is limited by the unknown higher-order QCD corrections, followed by the PDF uncertainties. A related extraction from the three-jet mass distribution, $d\sigma/dM_3$, has also been reported by CMS. For this measurement, the preliminary result is

$$\alpha_s(M_Z) = 0.1160^{+0.0025}_{-0.0023} (\text{exp, PDF, NP})^{+0.0068}_{-0.0021} (\text{scale}),$$

which is again dominated by scale variations. All these values of $\alpha_s(M_Z)$ reported by CMS are consistent, within uncertainties, with the PDG average value.

^dResults obtained using CT10 and MSTW08 are consistent with those using NNPDF2.1. However, if the ABM11 set is used, the extracted value of $\alpha_s(M_Z)$ is larger, with central value $\alpha_s(M_Z) = 0.1214$.

Turning to the results based on the ATLAS data, closely related to the CMS analysis based on $R_{3/2}$, a measurement of α_s from the ratios of three-jet over two-jet events has been presented by the ATLAS collaboration⁶⁹ from their 2010 7 TeV data (that is, based on only $\sim 40 \text{ pb}^{-1}$ of data). In this analysis, jets are clustered with the anti- k_T algorithm with $R = 0.6$, and only jets with $p_T \geq 60 \text{ GeV}$ and $|y| \leq 2.8$ are included. Two ratios are defined, the first as a function of the transverse momentum of the leading jet in the event and the second as a function of all the jets in the event, as follows:

$$R_{3/2}(p_T^{\text{lead}}) \equiv \frac{d\sigma_{N_{\text{jet}} \geq 3}/dp_T^{\text{lead}}}{d\sigma_{N_{\text{jet}} \geq 2}/dp_T^{\text{lead}}}, \quad R_{3/2}(p_T^{\text{all jets}}) \equiv \frac{\sum_i^{N_{\text{jets}}} d\sigma_{N_{\text{jet}} \geq 3}/dp_T^i}{\sum_i^{N_{\text{jets}}} d\sigma_{N_{\text{jet}} \geq 2}/dp_T^i}.$$

These two ratios have a direct sensitivity to α_s , of comparable size. The result of this analysis is

$$\alpha_s(M_Z) = 0.111 \pm 0.006 (\text{exp}) \quad {}^{+0.016}_{-0.003} (\text{theory}).$$

The uncertainty in the result is again dominated by the theoretical uncertainties due to scale variations, followed by the large experimental uncertainties: recall that this measurement is based on the 2010 dataset, with much less statistics than the CMS $R_{3/2}$ measurement.

Still with the 2010 7 TeV data, the ATLAS inclusive jet cross-sections were used by Malaescu and Starovoitov in Ref.⁷⁰ to perform an extraction of $\alpha_s(M_Z)$. The result reads:

$$\begin{aligned} \alpha_s(M_Z) = & 0.1151 \pm 0.0001 (\text{stat}) \pm 0.0047 (\text{sys}) \pm 0.0014 (p_T \text{ range}) \\ & \pm 0.0060 (\text{jet size}) \quad {}^{+0.0044}_{-0.0011} (\text{scale}) \quad {}^{+0.0022}_{-0.0015} (\text{PDF choice}) \\ & \pm 0.0010 (\text{PDF eig}) \quad {}^{+0.0009}_{-0.0034} (\text{NP corrections}). \end{aligned} \quad (1)$$

In this analysis the dominant systematic uncertainty was found to arise from the difference in the results obtained if either data with jet radius $R = 0.4$ or $R = 0.6$ is used, followed by the experimental systematics (dominated by the jet energy scale) and the unknown perturbative higher orders.

These results for the various determinations of the strong coupling from LHC jet data have been collected in Table 1, and summarized graphically in Fig. 7, together with other determinations from collider jet data from HERA and the Tevatron. In each case the plot shows the total uncertainty band, and the results of the individual determinations are compared with the PDG global average. The nice consistency of the determinations based on LHC data with the PDG average is clear from the plot. For completeness, Fig. 7 also includes the results of other determinations of $\alpha_s(M_Z)$ from jet data of non-LHC experiments, like H1, ZEUS, CDF and D0. It should be emphasized again that all these results are based on NLO QCD calculations, and therefore one expects substantial improvements once the same data is reanalyzed using the full NNLO results for jet production.^e

^e As an example, the data on hadronic jet shapes in electron-positron collisions from LEP was

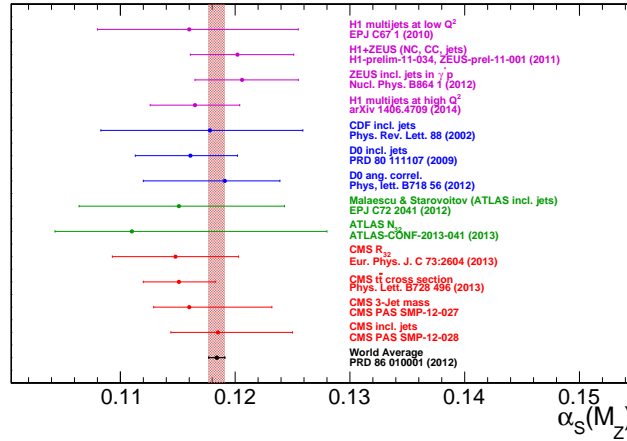


Fig. 7. Summary of recent determinations of the strong coupling constant from collider jet data, together with the PDG world average. In each case, the central value and the total uncertainty band are shown. The plot includes results from HERA and Tevatron together with the LHC determinations based on ATLAS and CMS data. Summary plot taken from the CMS Standard Model Twiki, <https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSMP>.

Table 1. Summary of direct determinations of the strong coupling constant $\alpha_s(M_Z)$ from jet measurements at the LHC. See text for more details of each specific analysis. In various cases, $\alpha_s(Q)$ has also been extracted at different scales Q without assuming the SM running.

Reference	Input measurement	$\alpha_s(M_Z)$
Ref. ⁷⁰	ATLAS 2010 incl jets	0.1151 ± 0.0001 (stat) ± 0.0047 (sys) ± 0.0014 (p_T range) ± 0.0060 (jet size) $+0.0044$ (scale) $+0.0022$ (PDF choice) ± 0.0010 (PDF eig) $+0.0009$ (NP corrections) -0.0034
Ref. ⁶⁹	ATLAS 2010 $R_{3/2}$	0.111 ± 0.006 (exp) $+0.016$ (theory) -0.003
Ref. ⁴⁷	CMS 2011 $R_{3/2}$	0.1148 ± 0.0014 (exp) ± 0.0018 (PDF) ± 0.0050 (th)
Ref. ²⁹	CMS 2011 inclusive jets	0.1185 ± 0.0019 (exp) ± 0.0028 (PDF) ± 0.0004 (NP) $+0.0053$ (scale) -0.0024
Ref. ⁷²	CMS 2011 three-jet mass	$0.1160^{+0.0025}_{-0.0023}$ (exp, PDF, NP) $+0.0068$ (scale) -0.0021

Finally, it is worth mentioning that at the LHC there are other processes, other than jet production, that can be used to extract $\alpha_s(M_Z)$. One example is provided by the total top-quark production cross-section.¹⁹ As reported in,¹⁹ using the NNLO calculation²¹ and NNPDF2.3⁷³ as input, the CMS collaboration extracted $\alpha_s(M_Z)$ from their 7 TeV inclusive top quark pair production measurements, finding the

reanalyzed in⁷¹ once the full NNLO calculation became available.

following result:

$$\begin{aligned}\alpha_s(M_Z) &= 0.1151^{+0.0028}_{-0.0027} (\text{tot}) \\ &= 0.1151^{+0.0017}_{-0.0018} (\text{exp})^{+0.0013}_{-0.0011} (\text{PDF})^{+0.0009}_{-0.0008} (\text{scale}) \\ &\pm 0.0013 (m_t^{\text{pole}}) \pm 0.0008 (E_{\text{LHC}}).\end{aligned}\tag{2}$$

Note the substantial reduction on scale uncertainties as compared to the determinations from jet data in Table 1, as a consequence of the availability of NNLO calculations for this process. This result is also shown in the summary plot in Fig. 7. Results for the central value of $\alpha_s(M_Z)$ and the corresponding uncertainties are similar when other PDF sets are used, except for the ABM11 set which prefers instead a larger value, $\alpha_s(M_Z) = 0.1187$. Note that for ABM11 the central value of α_s from their NNLO fit is much smaller, $\alpha_s(M_Z) = 0.1134$.

6. Summary and outlook

In this review, I have presented an overview of the constraints on the parton distributions of the proton and on the strong coupling constant α_s that have been obtained up to now from jet production measurements at the Large Hadron Collider. I have summarized various analyses that coincide qualitatively: LHC jet data provides important constraints on the medium and large- x gluon PDF, as well as on the large- x quarks. I have also discussed recent progress in theoretical calculations and tools for fitting jet data, and presented a possible strategy to include jet data in NNLO fits based on the use of approximate NNLO calculations from threshold resummation, validated with the exact NNLO calculations. These studies have so far been restricted to 7 TeV data; once the full Run I data is analyzed one expects more stringent constraints thanks to the increase in statistics, the corresponding improvement in systematic uncertainties and the extended lever arm in jet transverse momentum and dijet invariant mass. In the medium term, the higher-energy Run II will also provide a wide range of jet measurements with PDF sensitivity. In the analysis of Run II data, accounting for NLO electroweak corrections will be mandatory, since in the TeV region these can be comparable to QCD effects. As an illustration, CMS already includes these electroweak effects in their PDF and α_s study from the 2011 data,²⁹ and the same is true for ATLAS in their dijet measurements from the 2011 run.²⁶

I have then reviewed existing determinations of $\alpha_s(M_Z)$ using LHC jet data, and shown that, while experimental uncertainties are typically competitive with other processes, the overall precision is degraded by the lack of knowledge of the full NNLO calculation. Restricted to NLO theory, all these extractions are so far consistent within uncertainties with the current global PDG average. It is worth emphasizing that thanks to the LHC data the first direct measurements of $\alpha_s(Q)$ in the TeV region have been obtained, which provide for the first time important model-independent tests of the running of the coupling and search for possible deviations that could arise in New Physics scenarios. It will be important to reanalyze

these existing determinations once the full NNLO result is available, when scale uncertainties will decrease substantially. In addition, now that LHC jet data is also being included in most global PDF fits, it will be interesting to study what is the impact of available and future measurements into the determinations of α_s in the framework of global PDF fits. Once the Run II data is available, the kinematical coverage for high-precision direct extractions of the coupling constant will extend deep into the TeV region, thus it will provide a unique opportunity to search and exclude robust model-independent constraints on new colored matter sectors.

All in all, the LHC is providing a unique window to study in great detail the richness of the strong interaction, and jet production in particular offers a unique opportunity to pin down the parton distribution functions at large- x , a crucial prerequisite for New Physics searches. Remarkably, with the LHC we can directly extract for the first time α_s in the TeV region, validate if its running with the scale is consistent with the SM predictions and provide robust model-independent bounds on new colored sectors.

Acknowledgments

I am grateful to Gunther Dissertori for the invitation to write this review and to Stefano Carrazza, Stefano Forte for their suggestions and to Amanda Cooper-Sarkar, Sasha Glazov, Claire Gwenlan, Panos Kokkas, Klaus Rabbertz and Alessandro Tricoli for their comments on the manuscript and for information on the ATLAS and CMS jet measurements. This work has been supported by an STFC Rutherford Fellowship ST/K005227/1 and by the European Research Council with the Starting Grant "PDF4BSM".

References

1. Y. L. Dokshitzer, *Sov. Phys. JETP* **46**, 641 (1977).
2. G. Altarelli and G. Parisi, *Nucl. Phys.* **B126**, 298 (1977).
3. V. N. Gribov and L. N. Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 (1972).
4. H1 and ZEUS Collaborations (F. Aaron *et al.*), *JHEP* **1001**, 109 (2010), [arXiv:0911.0884 \[hep-ex\]](#), doi:10.1007/JHEP01(2010)109.
5. H1 and ZEUS Collaborations (H. Abramowicz *et al.*) (2015), [arXiv:1506.06042 \[hep-ex\]](#).
6. CDF Collaboration (F. Abe *et al.*), *Phys. Rev. Lett.* **62**, 613 (1989), doi:10.1103/PhysRevLett.62.613.
7. CDF Collaboration (F. Abe *et al.*), *Phys. Rev. Lett.* **68**, 1104 (1992), doi:10.1103/PhysRevLett.68.1104.
8. D0 Collaboration (B. Abbott *et al.*), *Phys. Rev. Lett.* **82**, 2451 (1999), [arXiv:hep-ex/9807018 \[hep-ex\]](#), doi:10.1103/PhysRevLett.82.2451.
9. A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, *Phys. Lett.* **B604**, 61 (2004), [arXiv:hep-ph/0410230](#), doi:10.1016/j.physletb.2004.10.040.
10. H. L. Lai, J. Huston, S. Kuhlmann, F. I. Olness, J. F. Owens, D. E. Soper, W. K. Tung and H. Weerts, *Phys. Rev.* **D55**, 1280 (1997), [arXiv:hep-ph/9606399 \[hep-ph\]](#), doi:10.1103/PhysRevD.55.1280.

11. CDF Collaboration (T. Aaltonen *et al.*), *Phys. Rev.* **D78**, 052006 (2008), [arXiv:0807.2204 \[hep-ex\]](#), doi:10.1103/PhysRevD.78.052006.
12. D0 Collaboration (V. M. Abazov *et al.*), *Phys. Rev. Lett.* **101**, 062001 (2008), [arXiv:0802.2400 \[hep-ex\]](#), doi:10.1103/PhysRevLett.101.062001.
13. NNPDF Collaboration (R. D. Ball, V. Bertone, S. Carrazza, C. S. Deans, L. Del Debbio *et al.*), *Nucl. Phys.* **B867**, 244 (2013), [arXiv:1207.1303 \[hep-ph\]](#), doi:10.1016/j.nuclphysb.2012.10.003.
14. A. D. Martin, W. J. Stirling, R. S. Thorne and G. Watt, *Eur. Phys. J.* **C63**, 189 (2009), [arXiv:0901.0002 \[hep-ph\]](#), doi:10.1140/epjc/s10052-009-1072-5.
15. J. Gao, M. Guzzi, J. Huston, H.-L. Lai, Z. Li *et al.*, *Phys. Rev.* **D89**, 033009 (2014), [arXiv:1302.6246 \[hep-ph\]](#), doi:10.1103/PhysRevD.89.033009.
16. J. Rojo *et al.*, *J. Phys.* **G42**, 103103 (2015), [arXiv:1507.00556 \[hep-ph\]](#), doi:10.1088/0954-3899/42/10/103103.
17. D. d'Enterria and J. Rojo, *Nucl. Phys.* **B860**, 311 (2012), [arXiv:1202.1762 \[hep-ph\]](#).
18. M. Czakon, M. L. Mangano, A. Mitov and J. Rojo, *JHEP* **1307**, 167 (2013), [arXiv:1303.7215 \[hep-ph\]](#), doi:10.1007/JHEP07(2013)167.
19. CMS Collaboration (S. Chatrchyan *et al.*), *Phys. Lett.* **B728**, 496 (2014), [arXiv:1307.1907 \[hep-ex\]](#), doi:10.1016/j.physletb.2014.08.040, 10.1016/j.physletb.2013.12.009.
20. S. A. Malik and G. Watt, *JHEP* **1402**, 025 (2014), [arXiv:1304.2424 \[hep-ph\]](#), doi:10.1007/JHEP02(2014)025.
21. M. Czakon, P. Fiedler and A. Mitov, *Phys. Rev. Lett.* **110**, 252004 (2013), [arXiv:1303.6254 \[hep-ph\]](#), doi:10.1103/PhysRevLett.110.252004.
22. R. Gauld, J. Rojo, L. Rottoli and J. Talbert (2015), [arXiv:1506.08025 \[hep-ph\]](#).
23. O. Zenaiev, A. Geiser, K. Lipka, J. Blmlein, A. Cooper-Sarkar *et al.* (2015), [arXiv:1503.04581 \[hep-ph\]](#).
24. M. L. Mangano, M. Moretti, F. Piccinini, R. Pittau and A. D. Polosa, *JHEP* **07**, 001 (2003), [arXiv:hep-ph/0206293](#).
25. CMS Collaboration (V. Khachatryan *et al.*) (2014), [CMS-PAS-SMP-14-002](#).
26. ATLAS Collaboration (G. Aad *et al.*), *JHEP* **1405**, 059 (2014), [arXiv:1312.3524 \[hep-ex\]](#), doi:10.1007/JHEP05(2014)059.
27. The NNPDF Collaboration (R. D. Ball *et al.*), *Nucl. Phys.* **B849**, 296 (2011), [arXiv:1101.1300 \[hep-ph\]](#).
28. F. Demartin, S. Forte, E. Mariani, J. Rojo and A. Vicini, *Phys. Rev.* **D82**, 014002 (2010), [arXiv:1004.0962 \[hep-ph\]](#), doi:10.1103/PhysRevD.82.014002.
29. CMS Collaboration (V. Khachatryan *et al.*), *Eur. Phys. J.* **C75**, 288 (2015), [arXiv:1410.6765 \[hep-ex\]](#), doi:10.1140/epjc/s10052-015-3499-1.
30. S. D. Ellis, Z. Kunszt and D. E. Soper, *Phys. Rev. Lett.* **64**, 2121 (1990), doi:10.1103/PhysRevLett.64.2121.
31. Z. Nagy, *Phys. Rev.* **D68**, 094002 (2003), [arXiv:hep-ph/0307268](#), doi:10.1103/PhysRevD.68.094002.
32. Z. Nagy, *Phys. Rev. Lett.* **88**, 122003 (2002), [arXiv:hep-ph/0110315 \[hep-ph\]](#), doi:10.1103/PhysRevLett.88.122003.
33. T. Carli, D. Clements, A. Cooper-Sarkar, C. Gwenlan, G. P. Salam *et al.*, *Eur. Phys. J.* **C66**, 503 (2010), [arXiv:0911.2985 \[hep-ph\]](#), doi:10.1140/epjc/s10052-010-1255-0.
34. fastNLO Collaboration (M. Wobisch, D. Britzger, T. Kluge, K. Rabbertz and F. Stober) (2011), [arXiv:1109.1310 \[hep-ph\]](#).
35. V. Bertone, R. Frederix, S. Frixione, J. Rojo and M. Sutton, *JHEP* **1408**, 166 (2014), [arXiv:1406.7693 \[hep-ph\]](#), doi:10.1007/JHEP08(2014)166.
36. J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni *et al.*, *JHEP* **1407**, 079

- (2014), [arXiv:1405.0301 \[hep-ph\]](#), doi:10.1007/JHEP07(2014)079.
37. A. Gehrmann-De Ridder, T. Gehrmann and E. W. N. Glover, *JHEP* **09**, 056 (2005), [arXiv:hep-ph/0505111 \[hep-ph\]](#), doi:10.1088/1126-6708/2005/09/056.
 38. A. Gehrmann-De Ridder, T. Gehrmann, E. Glover and J. Pires, *Phys.Rev.Lett.* **110**, 162003 (2013), [arXiv:1301.7310 \[hep-ph\]](#), doi:10.1103/PhysRevLett.110.162003.
 39. J. Currie, A. Gehrmann-De Ridder, E. Glover and J. Pires, *JHEP* **1401**, 110 (2014), [arXiv:1310.3993 \[hep-ph\]](#), doi:10.1007/JHEP01(2014)110.
 40. D. de Florian, P. Hinderer, A. Mukherjee, F. Ringer and W. Vogelsang, *Phys.Rev.Lett.* **112**, 082001 (2014), [arXiv:1310.7192 \[hep-ph\]](#), doi:10.1103/PhysRevLett.112.082001.
 41. N. Kidonakis and J. Owens, *Phys. Rev.* **D63**, 054019 (2001), [arXiv:hep-ph/0007268](#), doi:10.1103/PhysRevD.63.054019.
 42. M. C. Kumar and S.-O. Moch, *Phys. Lett.* **B730**, 122 (2014), [arXiv:1309.5311 \[hep-ph\]](#), doi:10.1016/j.physletb.2014.01.034.
 43. S. Carrazza and J. Pires, *JHEP* **1410**, 145 (2014), [arXiv:1407.7031 \[hep-ph\]](#), doi:10.1007/JHEP10(2014)145.
 44. NNPDF Collaboration (R. D. Ball *et al.*), *JHEP* **1504**, 040 (2015), [arXiv:1410.8849 \[hep-ph\]](#), doi:10.1007/JHEP04(2015)040.
 45. S. Dittmaier, A. Huss and C. Speckner, *JHEP* **1211**, 095 (2012), [arXiv:1210.0438 \[hep-ph\]](#), doi:10.1007/JHEP11(2012)095.
 46. ATLAS Collaboration (G. Aad *et al.*), *Phys. Rev.* **D86**, 014022 (2012), [arXiv:1112.6297 \[hep-ex\]](#).
 47. CMS Collaboration (S. Chatrchyan *et al.*), *Eur.Phys.J.* **C73**, 2604 (2013), [arXiv:1304.7498 \[hep-ex\]](#), doi:10.1140/epjc/s10052-013-2604-6.
 48. M. L. Mangano and J. Rojo, *JHEP* **1208**, 010 (2012), [arXiv:1206.3557 \[hep-ph\]](#), doi:10.1007/JHEP08(2012)010.
 49. ATLAS Collaboration (G. Aad *et al.*), *JHEP* **1502**, 153 (2015), [arXiv:1410.8857 \[hep-ex\]](#), doi:10.1007/JHEP02(2015)153.
 50. ATLAS Collaboration (G. Aad *et al.*), *Eur. Phys. J.* **C75**, 228 (2015), [arXiv:1411.1855 \[hep-ex\]](#), doi:10.1140/epjc/s10052-015-3363-3.
 51. CMS Collaboration (S. Chatrchyan *et al.*), *Phys.Rev.* **D87**, 112002 (2013), [arXiv:1212.6660 \[hep-ex\]](#), doi:10.1103/PhysRevD.87.112002.
 52. CMS Collaboration (V. Khachatryan *et al.*) (2013), CMS-PAS-SMP-12-012.
 53. CMS Collaboration (S. Chatrchyan *et al.*), *Phys.Rev.* **D90**, 072006 (2014), [arXiv:1406.0324 \[hep-ex\]](#), doi:10.1103/PhysRevD.90.072006.
 54. M. Cacciari, J. Rojo, G. P. Salam and G. Soyez, *JHEP* **12**, 032 (2008), [arXiv:0810.1304 \[hep-ph\]](#), doi:10.1088/1126-6708/2008/12/032.
 55. M. Dasgupta, L. Magnea and G. P. Salam, *JHEP* **02**, 055 (2008), [arXiv:0712.3014 \[hep-ph\]](#), doi:10.1088/1126-6708/2008/02/055.
 56. B. Watt, P. Motylinski and R. Thorne, *Eur.Phys.J.* **C74**, 2934 (2014), [arXiv:1311.5703 \[hep-ph\]](#), doi:10.1140/epjc/s10052-014-2934-z.
 57. L. Harland-Lang, A. Martin, P. Motylinski and R. Thorne, *Eur.Phys.J.* **C75**, 204 (2015), [arXiv:1412.3989 \[hep-ph\]](#), doi:10.1140/epjc/s10052-015-3397-6.
 58. S. Alekhin, J. Blumlein and S. Moch, *JHEP* **02**, 041 (2014), [arXiv:1211.2642 \[hep-ph\]](#), doi:10.1007/JHEP02(2014)041.
 59. G. Watt, *JHEP* **1109**, 069 (2011), [arXiv:1106.5788 \[hep-ph\]](#), doi:10.1007/JHEP09(2011)069.
 60. H.-L. Lai *et al.*, *Phys. Rev.* **D82**, 054021 (2010), [arXiv:1004.4624 \[hep-ph\]](#), doi:10.1103/PhysRevD.82.054021.
 61. S. Dimopoulos, S. Raby and F. Wilczek, *Phys. Rev.* **D24**, 1681 (1981), doi:10.1103/

- PhysRevD.24.1681.
62. D. Becciolini, M. Gillioz, M. Nardecchia, F. Sannino and M. Spannowsky, *Phys.Rev.* **D91**, 015010 (2015), [arXiv:1403.7411 \[hep-ph\]](#), doi:10.1103/PhysRevD.91.015010.
 63. E. L. Berger, M. Guzzi, H.-L. Lai, P. M. Nadolsky and F. I. Olness, *Phys. Rev.* **D82**, 114023 (2010), [arXiv:1010.4315 \[hep-ph\]](#), doi:10.1103/PhysRevD.82.114023.
 64. E. L. Berger, P. M. Nadolsky, F. I. Olness and J. Pumplin, *Phys. Rev.* **D71**, 014007 (2005), [arXiv:hep-ph/0406143 \[hep-ph\]](#), doi:10.1103/PhysRevD.71.014007.
 65. R. D. Ball, V. Bertone, L. Del Debbio, S. Forte, A. Guffanti *et al.*, *Phys.Lett.* **B707**, 66 (2012), [arXiv:1110.2483 \[hep-ph\]](#), doi:10.1016/j.physletb.2011.11.053.
 66. R. Thorne and G. Watt, *JHEP* **1108**, 100 (2011), [arXiv:1106.5789 \[hep-ph\]](#), doi:10.1007/JHEP08(2011)100.
 67. Particle Data Group Collaboration (K. A. Olive *et al.*), *Chin. Phys.* **C38**, 090001 (2014), doi:10.1088/1674-1137/38/9/090001.
 68. S. Alekhin, J. Blumlein, S. Klein and S. Moch, *Phys. Rev.* **D81**, 014032 (2010), [arXiv:0908.2766 \[hep-ph\]](#), doi:10.1103/PhysRevD.81.014032.
 69. ATLAS Collaboration (G. Aad *et al.*) (2013), [ATLAS-CONF-2013-041](#).
 70. B. Malaescu and P. Starovoitov, *Eur. Phys. J.* **C72**, 2041 (2012), [arXiv:1203.5416 \[hep-ph\]](#), doi:10.1140/epjc/s10052-012-2041-y.
 71. G. Dissertori, A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, G. Heinrich and H. Stenzel, *JHEP* **02**, 040 (2008), [arXiv:0712.0327 \[hep-ph\]](#), doi:10.1088/1126-6708/2008/02/040.
 72. CMS Collaboration (V. Khachatryan *et al.*), *Eur. Phys. J.* **C75**, 186 (2015), [arXiv:1412.1633 \[hep-ex\]](#), doi:10.1140/epjc/s10052-015-3376-y.
 73. R. D. Ball, S. Carrazza, L. Del Debbio, S. Forte, J. Gao *et al.*, *JHEP* **1304**, 125 (2013), [arXiv:1211.5142 \[hep-ph\]](#), doi:10.1007/JHEP04(2013)125.